GATUNDU FORM 4 EVALUATION EXAM
121/1
MATHEMATICS
PAPER I
JULY/AUGUST 2015
TIME: 2 ½ HOURS

SECTION I
ANSWER ALL QUESTIONS

1. Without using Logarithms tables or a calculator evaluate.

\[ \sqrt{\frac{384.16 \times 0.0625}{96.04}} \] 

(3 marks)

2. Simplify completely

\[ \frac{2x^2 - 98}{3x^2 - 16x - 35} \div \frac{x + 7}{3x + 4} \]

(4 marks)

3. Solve the following inequality and show your solution on a number line.

\[ 4x - 3 \leq \frac{1}{2} (x + 8) < x + 5 \]

(3 marks)

4. Rose bought a golden necklace for ksh.6000 and sold it to Betty at a loss of 30%.
Betty later sold it at a profit of 20%. What was Betty’s selling price.

(2 marks)

5. If \( x = \frac{2}{3} \) is a root of \( 6x^2 + kx - 2 = 0 \), find the value of \( k \) and the other root.

(4 marks)

6. Tap A takes 4 minutes to fill a tank and tap B takes 6 minutes to empty the tank. If
the tank has a capacity of 3000 litres find the volume of the tank after 2 minutes
when both taps are open.

(3 marks)

7. From a viewing tower 30 metres above the ground, the angle of depression of an
object on the ground is 30° and the angle of elevation of an aircraft vertically above
the object is 42°. Calculate the height of the aircraft above the ground.

(3 marks)
8. Find the equation of the perpendicular bisector of line AB where A is (3, 9) and B(7,5) giving your answer in the form ax + by + c = 0

9. Solve the simultaneous equations.

\[
\begin{align*}
xy &= 4 \\
x + y &= 5
\end{align*}
\]

10. Vectors A and B are \(2\mathbf{i} + 5\mathbf{i}\) and \(8\mathbf{i} - 7\mathbf{j}\) respectively. Find the coordinates of M which divide AB in the ratio 1:2.

11. Ruto is 12 years old. In three years time he will be \(\frac{1}{3}\) of his father’s present age. How old was his father 12 years ago.

12. Given \(a:b = 6:7\) and \(b:c = 14:17\) find \(a:b:c\).

13. The figure below is a velocity time graph for a car.

![Velocity time graph](image)

a) Find the total distance traveled by the car.

b) Calculate the deceleration of the car.

14. Two sides of a triangular piece of land are 21km and 32km long. If its area is \(240\text{km}^2\), find the obtuse angle between the sides.

15. Evaluate using square root, reciprocal and square tables only.
16. A cylinder of diameter 28m was drilled right through it as shown below. Calculate its surface area.

![Diagram of a cylinder with a hole drilled through it]

17. A bus left Makindu at 11.45 a.m and traveled towards Mombasa at an average speed of 80km/h. A Nissan Matatu left Makindu at 1.15 p.m on the same day and traveled along the same road at an average speed of 120km/hr. The distance between Makindu and Mombasa is 400km.

a) Determine the time of the day the Nissan overtook the bus. (5 marks)

b) Both vehicles continue towards Mombasa at their original speeds. Find how long the Matatu had to wait at Mombasa before the bus arrived. (5 marks)

18. Given that \( y = 7 + 3x - x^2 \), complete the table below

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-11</td>
<td></td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-11</td>
</tr>
</tbody>
</table>

b) On the grid provided and using a suitable scale draw the graph of \( y = 7 + 3x - x^2 \). (3 marks)

c) On the same grid draw a straight line using the graph to solve \( x^2 - 4x - 3 = 0 \) (3 marks)

d) Determine the coordinates of the turning point of the curve. (2 marks)

19. From a reservoir, water flows through a cylindrical pipe of diameter 0.2m at a rate of 0.35m/s.
a) Determine the number of litres of water discharged from the reservoir in one hour. (4 marks)

b) The water flows from the reservoir for 18 hours per day for 25 days per month and serves a population of 2500 families. Determine the average consumption of water per family per month giving your answer to nearest 100 litres. (4 marks)

c) The water is charged at the rate of sh.450 per 100 litres. Calculate the average water bill per family per month. (2 marks)

20. A room is constructed such that its external length and breadth are 7.5m and 5.3m respectively. The thickness of the wall is 15cm and its height is 3.3 metres. A total space of 5m$^2$ is left for doors and windows on the walls.

a) Calculate the volume of:
   (i) the materials needed to construct the walls without the doors and windows. (4 marks)
   (ii) the materials needed to construct the walls with doors and windows. (2 marks)

b) The blocks used in constructing the walls are 450mm by 200mm by 150mm. 0.225m$^3$ of cement is used to join the blocks. Calculate the number of blocks. Calculate the number of blocks needed to construct the room. (4 marks)

21. Every Sunday, Chalo drives a distance of 80km on a bearing of 074° to pick up his brother Ben to go to church. The church is 75km from Ben’s house on a bearing of S50°E. After church they drive a distance of 100km on a bearing of 260° to check on their father before Chalo drives to Ben’s home to drop him off then proceeds to his house.

a) Using a scale of 1cm represent 10km show the relative positions of these places. (4 marks)

b) Use your diagram to determine
   (i) The true bearing of Charo’s
   (ii) The compass of bearing of the father’s home from Ben’s home (1 mark)
   (iii) The shortest distance between Ben’s home and father’s home. (2 marks)
   (iv) The total distance Charo travels’ every Sunday. (2 marks)
22. The following measurement were recorded in a field book using XY as the baseline. XY = 400m.

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>C60</td>
<td>340</td>
</tr>
<tr>
<td></td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>240</td>
</tr>
<tr>
<td></td>
<td>220</td>
</tr>
<tr>
<td>B100</td>
<td>140</td>
</tr>
<tr>
<td>A120</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

a) Using a scale of 1:4000 draw an accurate map of the farm. (4 marks)
b) Determine the actual area of the farm in hectares. (4 marks)
c) If the farm is on sale at sh.80,000 per hectare find how much the farm costs. (2 marks)

23. A tailor bought a number of suits at a cost of sh.57,000 from Ken-suit wholesalers. Had he bought the same number of suits from Umoja wholesalers it would have costed him sh.480 less per suit. This would have enabled him to buy 4 extra suits for the same amount of money.

a) Find the number of suits the tailor bought. (6 marks)
b) The tailor later sold each suit for sh.720 more than he had paid for it. Determine the percentage profit he made. (4 marks)

24. A particle P moves in a straight line such that t seconds after passing a fixed point Q. it’s velocity is given by the equation $2t^3 - 10t + 12$ find:

a) The values of t when p is instantaneously at rest. (2 marks)
b) An expression for the distance moved by P after t seconds. (2 marks)
c) The total distance traveled by P in the first 3 seconds after passing point O. (3 marks)
d) The distance of P from O when acceleration is zero. (3 marks)
SECTION I (50 MARKS) ANSWER ALL QUESTIONS

1. Use mathematical tables to evaluate. (4 marks)

\[
\sqrt{3 \times \frac{4\cos 60^\circ \times 0.1324^2}{5\log 7}}
\]

2. Solve for x in the equation (3 marks)

\[
\sin(4x - 10)^\circ - \cos(x + 60)^\circ = 0
\]

3. A radio cassette is offered for sale at shs 8,000 or a deposit of shs. 1,000 and 15 monthly repayments of shs 840. Find the rate of interest compounded monthly that is being charged under hire purchase terms. (4 marks)

4. A colony of insects was found to have 250 insects at the beginning. Thereafter the number of insects doubled every 2 days. Find how many insects there were after 16 days. (3 marks)

5. Under a shear with x-axis invariant a square with vertices A(1,0), B(3,0), C(3,2) and D(1,2) is mapped onto a parallelogram with vertices A'(1,0) B'(3,0), C'(7,2) and D'(5,2). Find the shear matrix. (3 marks)

6. Using a ruler and a pair of compasses only construct a triangle PQR in which QR is 6.6cm, P=3.8cm and PQ = 5.6cm. Locate point x inside triangle PQR which is equidistant from P and R such that \( \angle PXR = 90^\circ \). (3 marks)

7. Find the variance and standard deviation of 3, 5, 7, 9, 11 (3 marks)
8. P and Q are two points such that \( \mathbf{OP} = i + 2j + 3k \) and \( \mathbf{OQ} = 4i + 5j - 3k \). M is a point that divides PQ externally in the ratio 3:2. Find the co-ordinates of M. (3 marks)

9. The sector below has a radius of 12cm and an angle \( \angle AOC = 60^\circ \) is folded to form a cone. Find the volume of the cone formed. (4 marks)

10. Find the equation of the normal to the tangent of the curve \( y = x^3 - 3x^2 + 2x + 1 \) at the point where \( x = 3 \). Leave your answer in the form \( y = mx + c \). (3 marks)

11. Without using mathematical tables or calculator; evaluate: (3 marks)

\[
\frac{\cos 135^\circ - \sin 30^\circ}{\sin 135^\circ + \sin 30^\circ}
\]

12. Find the midpoint of the straight line joining A (2,1) and D (6,5). (2 marks)

13. The equation of a circle centre \( (h, k) \) is \( 2x^2 + 2y - 8x + 5y + 10 = 0 \). Find the values of \( h \) and \( k \). (3 marks)

14. Make \( y \) the subject of the formula given

\[
H = \sqrt{\frac{t}{q-y^2}}
\]

(3 marks)

15. If \( \frac{1}{a-2} - \frac{1}{a+2} = \frac{c}{a^2-b} \) for all values of \( a \), evaluate \( c \) and \( b \). (3 marks)

16. X and Y are two variables such that Y is partly constant and partly varies inversely as the square of X. If \( Y = 3 \) when \( X = 2 \) and \( Y = 5 \) when \( X = 1 \), find Y when \( X = 4 \).
17. The table below shows the number of students who scored marks in mathematics test.

<table>
<thead>
<tr>
<th>Marks</th>
<th>1-10</th>
<th>11-20</th>
<th>21-30</th>
<th>31-40</th>
<th>41-50</th>
<th>51-60</th>
<th>61-70</th>
<th>71-80</th>
<th>81-90</th>
<th>91-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>10</td>
<td>12</td>
<td>17</td>
<td>15</td>
<td>16</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>

a) Draw a cumulative frequency graph for the data. (4 marks)

b) Use the graph to estimate the median mark. (2 marks)

c) If students who score over 40 marks pass the tests estimate the percentage of the students

   i) who passed (2 marks)

   ii) who failed (2 marks)

18. In a geometrical progression, the sum of the second and third terms is 6; and the sum of the third and fourth terms is -12. Find:

   a) (i) The first term (3 marks)

      (ii) The common ration (3 marks)

   b) The sum of number of consecutive terms of an arithmetical progression is -19 ½ ; the first term is 16 ½ ; and the common difference is -3. Find the number of terms. (4 marks)

19. a) PQRS is a quadrilateral with vertices P(1, 4) Q(2, 1), R(2, 3) and S(6, 4). On the grid provided plot the quadrilateral. (1 mark)

   b) Draw P₁Q₁R₁S₁ the image of PQRS under a positive quarter turn about the origin and write down its co-ordinates. (3 marks)

   c) Draw P₁₁Q₁₁R₁₁S₁₁ the image of P₁Q₁R₁S₁ under the transformation whose matrix is 

      \[
      \begin{pmatrix}
      1 & 0 \\
      -2 & 1
      \end{pmatrix}
      \]

      and write down its co-ordinates. (3 marks)
d) Determine the matrix $T$ of a single transformation that maps $PQRS$ onto $P'^{11}Q'^{11}R'^{11}S'^{11}$

20. In the figure below, $E$ is the midpoint of $AB$, $OD:DB=2:3$ and $F$ is the point of intersection of $OE$ and $AD$.

![Figure](image)

a) Given that $OA = \mathbf{a}$ and $OB = \mathbf{b}$, express in terms of $a$ and $b$
   →

i) $\mathbf{OE}$

ii) $\mathbf{AD}$

b) Given further that $\mathbf{AF} = t\mathbf{AD}$ and $\mathbf{OF} = s\mathbf{OE}$ where $s$ and $t$ are scalars, find the values of $s$ and $t$.

(5 marks)

c) Show that $O$, $F$ and $E$ are collinear.

(3 marks)

21. The position of two towns $P$ and $Q$ are given to the nearest degrees as $P(45^\circ N, 110^\circ W)$ and $Q(45^\circ N, 70^\circ E)$ Take $\Pi= 3.142$, Radius of the earth $R = 6370$km. Find

a) The distance between the two towns along the parallel of latitude in km.

(3 marks)

b) The distance between the towns along a parallel of latitude in nautical miles.

(3 marks)

c) A plane flew from $P$ to $Q$ taking the shortest distance possible. It took the plane 15 hours to move from $P$ and $Q$. Calculate it’s speed in knots.

(4 marks)
22. Complete the table below (2 marks)

<table>
<thead>
<tr>
<th>θ</th>
<th>-180°</th>
<th>-150°</th>
<th>-120°</th>
<th>-90°</th>
<th>-60°</th>
<th>-30°</th>
<th>0°</th>
<th>30°</th>
<th>60°</th>
<th>90°</th>
<th>120°</th>
<th>150°</th>
<th>180°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y=sin(θ+30)°</td>
<td>-1</td>
<td>0.50</td>
<td>0.50</td>
<td>1.73</td>
<td>-1.73</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y=2cos(θ+30)°</td>
<td>0</td>
<td>1.73</td>
<td>-1.73</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) On the same axes draw the graphs of \( y= \sin(\theta+30)° \) and \( y= 2\cos(\theta+30)° \). (5 marks)

c) Use your graphs to solve the equation \( 2\cos(\theta+30)° - \sin(\theta+30)° = 0 \). (2 marks)

d) State the amplitude of each wave. (1 mark)

23. Two wheels have radii 20cm and 30cm. Their centres are 70cm apart. A belt, passes tightly round the wheels as shown below.

![Diagram of two wheels connected by a belt]

a) Calculate the length of AB and FE. (3 marks)

b) Evaluate the angles AOC and BCO. (3 marks)

c) Calculate the total length of the belt A B G E F H A. (4 marks)

24. Given the equations: \( y = 4 - x^2 \) and \( y = x^2 - 2x \);

a) Find the co-ordinates of the points where the two curves meet. (2 marks)

b) Find the co-ordinates of points where \( y=4 - x^2 \) meet:
   (i) The x-axis. (1 mark)
   (ii) The y-axis (1 mark)

c) Find the co-ordinates of the points where \( y=x^2 – 2x \) meet;
   (i) The x-axis (1 mark)
   (ii) The y-axis (1 mark)
d) Sketch the two curves above on the same axes (1 mark)

e) Find the area enclosed between the curves $y=4-x^2$ and $y=x^2 - 2x$. (3 marks)